

# Disorder effects in cellular automata for two-lane traffic

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For single-lane traffic models it is well known that particle disorder leads to platoon formation at low densities. Here we discuss the effect of slow cars in two-lane systems. Surprisingly, even a small number of slow cars can initiate the formation of platoons at low densities. The robustness of this phenomenon is investigated for different variants of the lane-changing rules as well as for different variants on the single-lane dynamics. It is shown that anticipation of drivers reduces the influence of slow cars drastically.

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## I. INTRODUCTION

In recent years it has turned out that cellular automata (CA) are excellent tools for the simulation of large scale traffic networks. The most prominent example for this kind of models has been introduced by Nagel and Schreckenberg [1] (NaSch model). Its properties have been discussed in detail in the past few years [2–4]. Also more sophisticated CA have been developed, e.g. models with so called ‘slow to start’ rules, which are able to reproduce hysteresis effects in traffic flow [5].

While most investigations consider homogeneous systems with one type of cars on a translational invariant lattice, real traffic is in many respects inhomogeneous. In general, different types of cars are present and from daily experience it is known that slow cars have strong influence on the systems performance. This is obviously most pronounced in single-lane traffic, where passing is not possible and therefore the slow cars dominate the dynamics. This intuitive picture has been confirmed analytically [6–9] for the asymmetric exclusion process, which is closely related to the NaSch model with  $v_{max} = 1$ , and numerically [10] for the NaSch model with  $v_{max} > 1$ . Beyond the basic mechanism of platoon formation behind the slowest car, it has been shown that a phase transition occurs, which is in some sense similar to the Bose-Einstein condensation [8]. For multi-lane models the influence of slow cars is not so obvious. Recently it has been shown, that already a small fraction of slow cars can dominate multi-lane systems at low densities [11], but nevertheless the effect of different types of cars in multi-lane traffic is far from being clarified.

In this work we show that for very small densities of slow cars platoon formation is observable, if one considers the CA model for two-lane traffic introduced by Rickert et al. [12]. Moreover we discuss alternative lane-changing rules as well as variants of the basic model.

For the sake of completeness we briefly recall the definition of the NaSch model [1]. The NaSch model is a discrete model for traffic flow. The road is divided into cells which can be either empty or occupied by a car with a velocity  $v = 0, 1, \dots, v_{max}$ . The cars move from the left to the right on a lane with periodic boundary conditions

and the system update is performed in parallel for all cars according to the following four rules:

1. Acceleration:  $v \rightarrow \min(v + 1, v_{max})$ .
2. Deceleration:  $v \rightarrow \min(v, gap)$
3. Noise:  $v \rightarrow \max(v - 1, 0)$  with probability  $p$ .
4. Motion:  $x \rightarrow x + v$ .

$v$  denotes the velocity,  $v_{max}$  the maximum velocity and  $x$  the position of a car,  $gap$  specifies the number of empty cells in front of the car.

In order to extend the model to multi-lane traffic one has to introduce lane-changing rules. This is usually done by dividing the update step into two sub-steps: In the first sub-step, cars may change lanes in parallel according to lane-changing rules and in the second sub-step the lanes are considered as independent single-lane NaSch models.

The lane-changing rules can be symmetric or asymmetric with respect to the lanes and to the cars. Rickert et al. [12] have assumed a symmetric rule set where cars change lanes if the following two criteria are fulfilled:

- Incentive criterion:
  1.  $v_{hope} > gap$ , with  $v_{hope} = \min(v + 1, v_{max})$ .
- Safety criteria:
  2.  $gap_{other} > gap$ .
  3.  $gap_{back} \geq v_{max}$ .

Here  $gap$ ,  $gap_{other}$ ,  $gap_{back}$  denote the number of free cells between the car and its predecessor on the actual lane and its two neighbor cars on the desired lane, respectively.

Recently, asymmetric rule sets have been proposed for the description of highway traffic on the german “Auto-bahn” where overtaking on the right lane is forbidden.

These rule sets are able to reproduce the lane-usage inversion<sup>1</sup> observed experimentally in excellent agreement with measurements [12–17,11].

The outline of this paper is as follows: In the next section the lane change behavior of a homogeneous NaSch model is considered. It will be shown that the lane-change probability decreases for certain braking parameters  $p$ . This leads to a total domination of the system not only by a fraction of slow cars, but also by just one slow car. Therefore we will first change the velocity update order and consider a sequential version of the NaSch model in section III. We also present a parallel “anticipation” model which shows features found in the basic parallel as well as in the sequential model. Finally, several other variants of the basic model are discussed briefly. In the last section a short summary and a discussion will follow.

## II. BASIC MODELS

As mentioned above it is necessary to choose an appropriate lane-changing rule set depending on the experimental situation. First we consider symmetric lane-changing rules, which are relevant for traffic in towns and on highways, where overtaking on both lanes is allowed.

Fig. 1 shows the fundamental diagram of a periodic two-lane system and the density dependence of the lane-changing frequency. The simulations reproduce well known results, e.g. an increase of the maximum flow per lane compared to the flow of a single-lane road. In addition, one finds surprising new results like a local minimum of the lane-changing frequency near the density of maximum flow for small braking probabilities  $p$ . Obviously jams can be partially avoided by changing the lanes and therefore the capacity of the system is slightly increased.

The behavior of the lane-changing frequency can be explained if one takes into account the number of empty cells necessary for a lane-changing procedure. Two prerequisites have to be fulfilled in order to initiate a lane change. First, the situation on the other lane must be more convenient and second, the safety rules must be fulfilled. Therefore one needs typically  $2v_{max} + 1$  empty cells on the destination lane for a lane-changing maneuver in the free flow regime (Fig. 2). Hence, one finds a local maximum of the lane-changing frequency near  $\rho_s = \frac{1}{2} \frac{1}{v_{max}+1}$  if the cars are ordered homogeneously, which typically happens for small values of  $p$ . For larger values of the braking noise (i.e.  $p = 0.5$ ) no local maximum is observable. Increasing the density for sufficiently small values of  $p$ , one finds a pronounced minimum of the lane changing frequency. This can be understood in

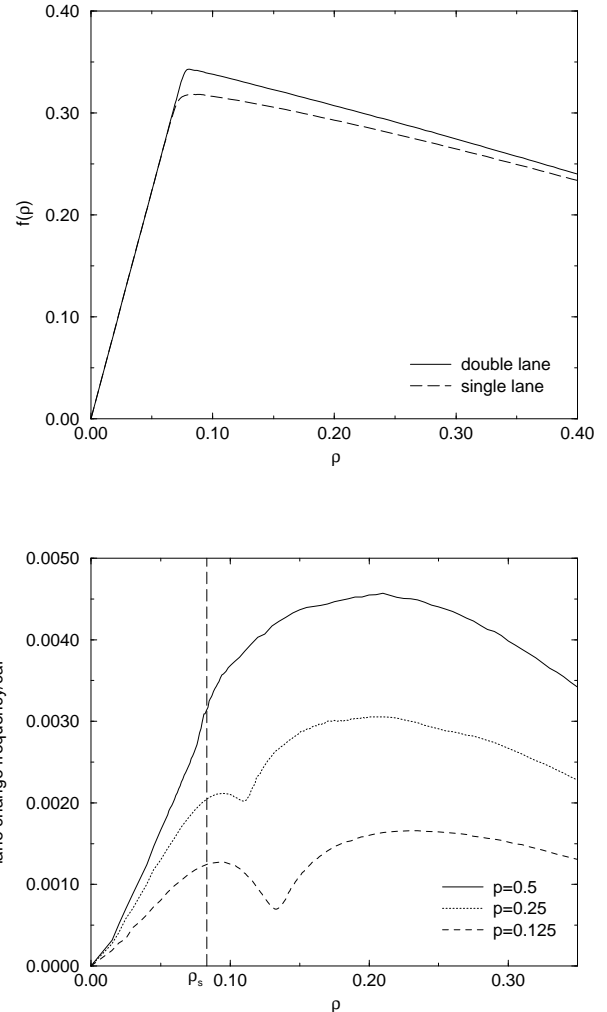


FIG. 1. Top: Flow per lane of the single lane model compared with the two lane model for systems with  $v_{max} = 5$  and  $p = 0.5$ . Bottom: Lane change frequency in the two lane model for different braking parameters  $p$ .

the limit  $p \rightarrow 0$  where, for  $\rho = \frac{1}{v_{max}+1}$ , the cars are perfectly ordered with a gap of  $v_{max}$  sites between consecutive vehicles. Obviously in this case both the incentive and the safety criteria are never fulfilled and the lanes are completely decoupled. For small noise the ordering mechanism is still present and therefore the number of lane changes is drastically reduced near  $\rho = \frac{1}{v_{max}+1}$ . For large values of  $p$  this kind of ordering is suppressed since fluctuations of the distance between consecutive cars become larger. Therefore large gaps become more likely than in the deterministic limit and lane changes become possible again. A further increase of the average density reduces the probability to find a gap on the other lane which is large enough for a lane change. Obviously the global maximum is reached at much higher values of  $\rho$ , because the incentive criterion is fulfilled for more and

<sup>1</sup>Above a certain density most of the cars are on the left lane.

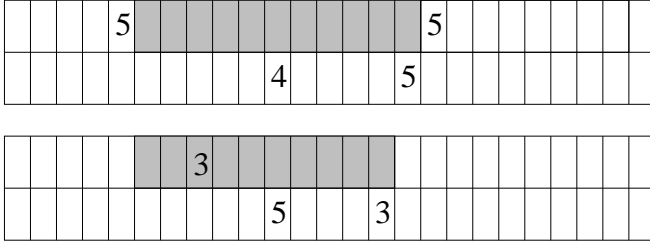


FIG. 2. Top: Necessary safety gap for a lane change in the free flow regime. Bottom: Possible positions of slow cars which lead to a plug. The cars are driving from left to right.

more cars at higher densities, and at intermediate densities large gaps are still present because the microscopic states are inhomogeneous.

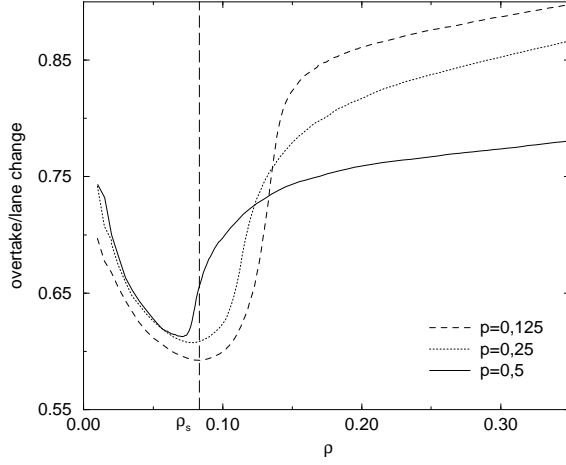


FIG. 3. Fraction of lane changes which are overtaking maneuvers for different braking parameters  $p$ .

This intuitive picture can be confirmed by measuring the number of “successful” lane changes, where a car actually overtakes its former predecessor (Fig. 3) in the next timestep. Overtaking means that following and leading car have change their rôle. Again we find a local minimum near  $\rho_s$ . For densities  $\rho > \rho_s$  the efficiency of lane changes increases monotonously, meaning that the cars can improve their average velocity due to changing the lane.

After this short review of the results for homogeneous systems, we now consider different types of cars which is obviously more relevant for practical purposes. As a first step towards realistic distributions of free flow velocities we have chosen two types of cars, e.g. slow cars (‘trucks’) with  $v_{max}^{slow} = 3$  and fast cars with  $v_{max}^{fast} = 5$ , analogous to Ref. [11]. The simulations have been carried out with 5% of slow cars, which are initially positioned randomly. The fast as well as the slow cars may use both lanes, e.g. both cars are treated equally with respect to the lane-changing behavior.

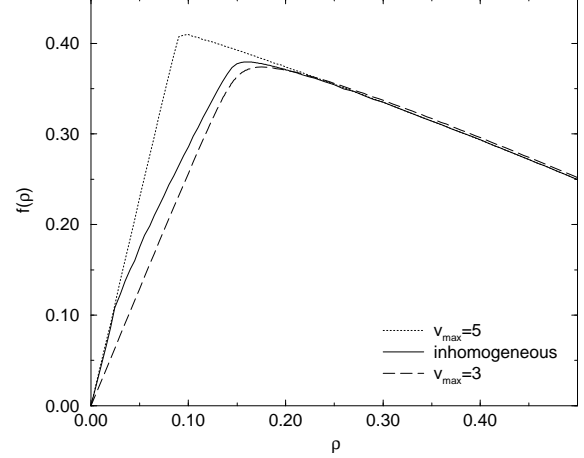


FIG. 4. Comparison of the flow per lane of the inhomogeneous model with the corresponding homogeneous models for  $p = 0.4$ .

In Fig. 4 the effects of the slow cars on the average flow of the two-lane system is compared with the fundamental diagram of a single lane road with one slow car. Since passing is not allowed for a single-lane system, clearly the slow car dominates the average flow at low densities and platoon formation is observable [6–10]. Surprisingly the two-lane system shows a quite similar behavior, although passing is allowed and the fraction of slow cars is rather small, which is consistent with the results of Ref. [11].

In [12] it was shown, that two trucks driving side by side can form a “plug” and blockade the following traffic. Similar observations have been made in [11,13,18]. For constituting such a plug it is not necessary that both cars are driving side by side. A fast car with velocity  $v = v_{max}^{slow}$  driving behind a slow car needs  $v_{max}^{slow} + 1$  empty cells in the driving direction and  $v_{max}^{fast}$  empty cells as safety gap on the other lane for a successful lane change. Therefore two trucks are able to form a plug even with a gap of 9 cells between them (Fig. 2).

These plugs lead to platoon formation analogous to a single-lane system. Obviously the average velocity of such platoons is limited by the free flow velocity of the slow cars. Therefore, if the plugs have long lifetimes, the total flow can not exceed the capacity of a homogeneous system of slow cars.

In fact, plug configurations are quite stable because the slow cars generically will have a large gap in front such that the incentive criterion hardly ever will be fulfilled. Therefore the leading vehicle of a platoon rarely changes the lane. Moreover both leading cars of the plug drive with the same average velocity and therefore larger distances between the slow cars only occur due to velocity fluctuations. This means that plugs have strong influence on the stationary state.

In many respects the situation is analogous to single-

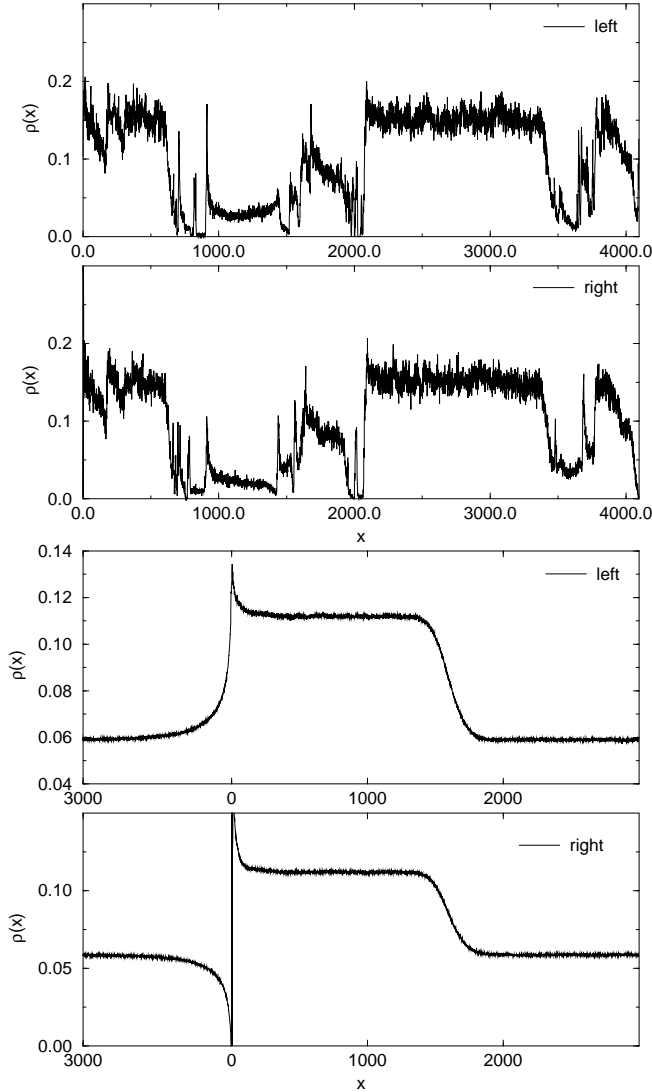


FIG. 5. Top: Density profile relative to a slow car on one lane in the inhomogeneous model (with 5% slow cars) at  $\rho = 0.1$  and  $p = 0.4$ . Bottom: Density profile of both lanes in a system with just one slow car relative to the slow car for  $\rho = 0.08$  and  $p = 0.125$ . The system size in both cases is  $L = 4096$ .

lane traffic. This can be exemplified by looking at the density profiles of both lanes relative to a slow car on one of the lanes. Obviously the flow is dominated by plugs. Beneath several small jams one big jam parallel on both lanes occurs, comparable to the density profile of a single-lane road (Fig. 5). Note, however, that in contrast to one-lane systems [6] the variance of the distance distribution does not diverge.

For densities  $\rho > \rho_{max}$ , where  $\rho_{max}$  is the density where the flow becomes maximal, also the slow cars will not always find a sufficient large gap in front. Therefore the average flow is limited by the number of empty cells but not by the low maximum velocity of the slow cars. Consequently we get the same flow for the homogeneous system of fast as well as for the inhomogeneous case.

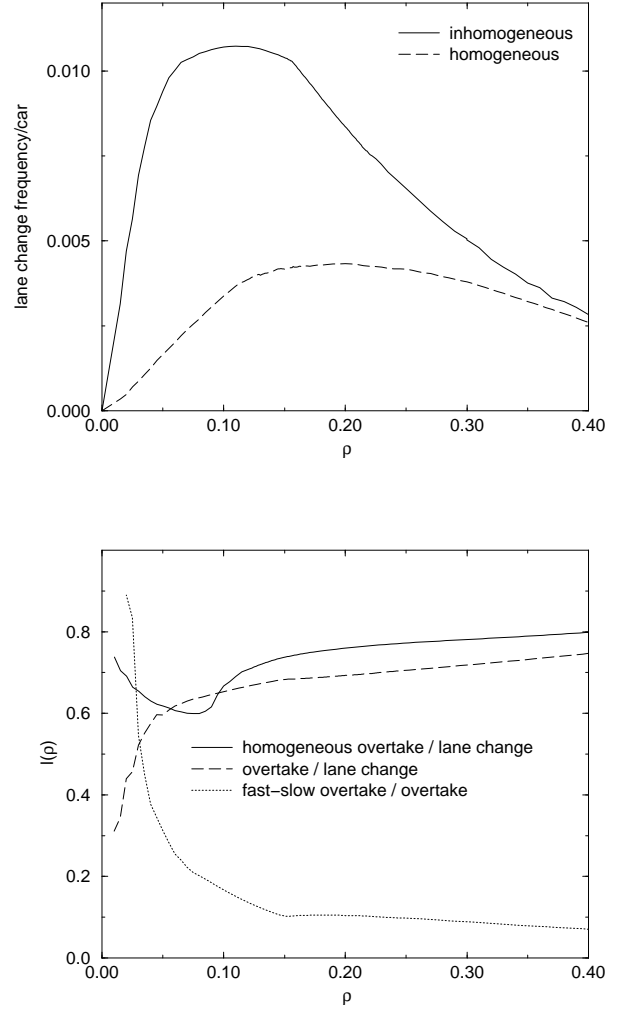


FIG. 6. Top: Lane-changing frequency of the inhomogeneous model compared to the corresponding homogeneous model for  $p = 0.4$ . Bottom: Fraction  $l(\rho)$  of lane changes which are overtaking maneuvers in the homogeneous (full line) and inhomogeneous system (broken) line. The dotted line shows the fraction of overtaking maneuvers where a slow car is passed by a fast car.  $p = 0.4$  in all cases.

Fig. 6 shows a comparison of the lane-changing frequency and the fraction of successful overtaking maneuvers for the homogeneous and the inhomogeneous models. For small densities the lane-changing frequency in the inhomogeneous model is increased drastically. For very small densities most of the overtaking maneuvers are those where fast cars pass a slow car. This corresponds to the regime where the trucks do not have a significant effect on the flow. With increasing density the fraction of fast-slow overtaking maneuvers decreases although the fraction of successful overtaking maneuvers increases.

As a next step we study the effect of a *single* slow car on the behavior of the system. This slow car with  $v_{max}^{slow} = 3$  always moves on the same (right) lane, i.e. it is

not allowed to change the lane. In contrast to the former example, one would expect that this slow car disturbs the system only locally and therefore should not have any effects on e.g. the fundamental diagram.

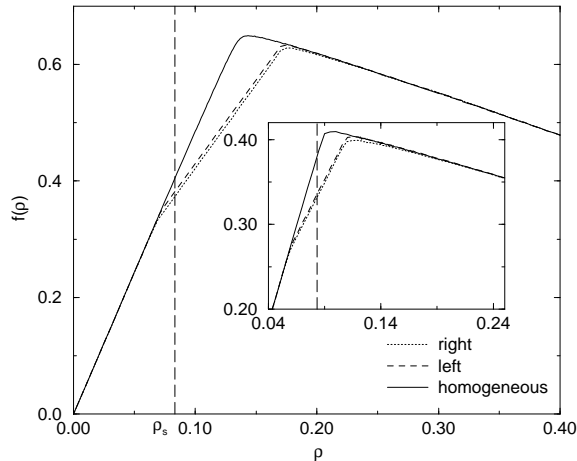


FIG. 7. Comparison of the flow per lane of a two-lane system with one slow car with the homogeneous system,  $p = 0.125$ . The inset shows the same situation for  $p = 0.4$ .

Simulations show, however, that above a certain density  $\rho_T$  already one slow car is sufficient to dominate the flow on both lanes (Fig. 7). The transition density  $\rho_T$  depends on the maximum velocity of the fast cars and the braking parameter  $p$ . While  $\rho < \rho_T$  the fast cars change to the left lane in order to avoid being trapped behind the slow car. With increasing density the probability for a lane change decreases and a jam behind the truck forms. This jam causes a jam on the left lane (Fig. 6). More and more cars change to the left and form a region of high density parallel to the jam. This region is disturbed if a jammed fast car can change to the left. The local defect, the slow car, causes a “parallel” jam on the left lane. Together with the truck this parallel jam behaves like a plug. Therefore the flow on both lanes is dominated by this plug for  $\rho > \rho_T$ , the slow car “synchronizes” the flow on both lanes. The effect of vehicles trapped behind the slow car can be illustrated by measurements of the waiting time distribution of the first fast car behind the slow one until it can change the lane (Fig. 8). Close to  $\rho_T$  the waiting time  $T$  jumps to macroscopic values.

The choice of a low braking parameter  $p$ , on the one hand, leads to a decrease of the lane change probability and therefore to an influence even on the homogeneous system. On the other hand, with increasing  $p$  interactions between the cars increase. A slow car can easily cause a jam and form a “plug”. Hence, the system is dominated by one slow car above a certain density  $\rho_T$  for a wide range of braking parameters  $p$ . With decreasing  $p$  one observes  $\rho_T \rightarrow \rho_s \approx \frac{1}{2} \frac{1}{v_{max} + 1}$ .

In order to weaken the influence of the slow cars one

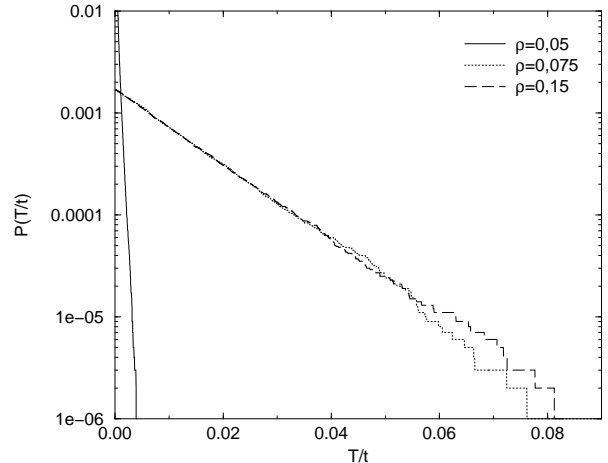


FIG. 8. Waiting time  $T$  distribution for  $p = 0.125$ .  $t$  denotes the measure time.

can introduce asymmetric lane-changing rules. This can be done by neglecting the first rule of the symmetric rule set of Rickert et al. [12] for the change from the left to the right lane. Now, the left lane is designated as the passing lane and cars are trying to change back to the right lane as soon as possible. In Fig. 9 the average flow of asymmetric models with  $v_{max} = 5$ ,  $v_{max} = 3$  and  $v_{max}^{fast} = 5$  with 5% slow cars ( $v_{max}^{slow} = 3$ ) is compared. For very low densities, fast cars can pass slow cars more effectively if asymmetric lane-changing rules are applied. Obviously, the flow can be increased for densities below  $\rho_{max}$  although the strong influence of the slow cars is still apparent.

With asymmetric rules almost all cars drive on the right lane at low densities (Fig. 9). Hence, the system is divided into a “fast” and a “slow” lane. Therefore at low densities there is only a low probability for plug formation of two slow cars driving side by side. With increasing density this probability and the probability for plugs formed by a slow car and its parallel jam increases, such that platoon formation occurs and the system is dominated by slow cars. Note that the density on the fast lane is still reduced in that regime, in contrast to realistic systems, where lane inversion is observable. Therefore the increase of the system’s performance is in some sense artificial.

The results presented up to now show that even very small densities of slow cars can dominate the behaviour of the whole system. Since this appears to be somewhat unrealistic we will investigate in the following several modifications of the basic rule set where the influence of slow cars is reduced.

### III. MODIFIED MODELS

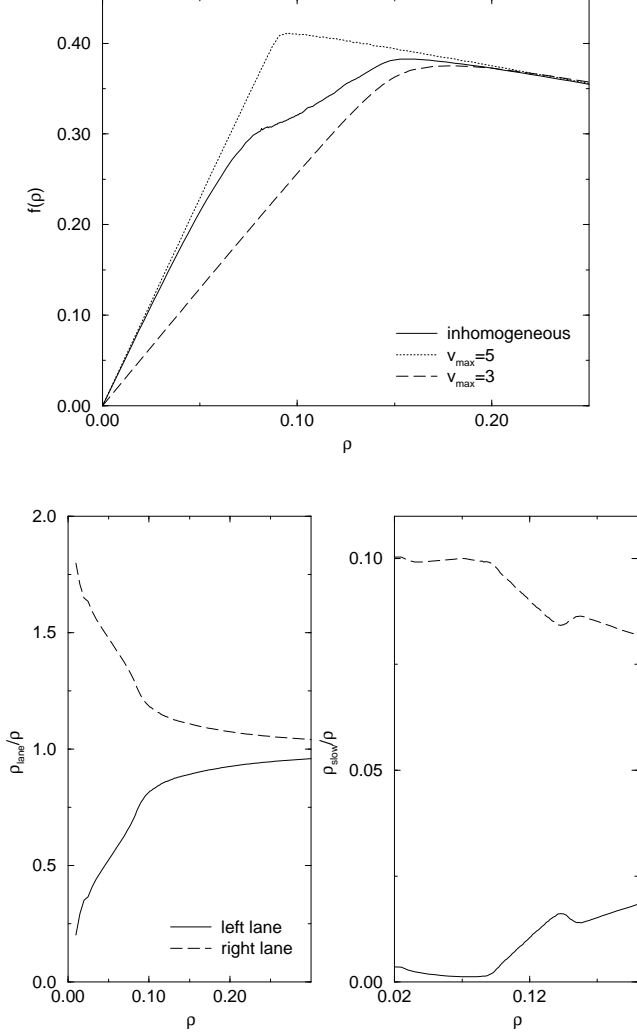


FIG. 9. Top: Average flow of different asymmetric models with  $p = 0.4$ . Bottom: Relative densities of the lanes and slow cars for a inhomogeneous model with 5% slow cars.

### A. Sequential update

The results of the previous section have shown, that the safety criteria demand large gaps between consecutive cars on the destination lane for lane-changing maneuvers. This leads to plug formation and therefore a drastic flow reduction by slow cars already in the free flow regime. Much smaller gaps would be sufficient for a lane change, if the driver takes into account the behavior of the predecessor in the next time step. Such anticipation effects can be most easily considered in a discrete model, if the update procedure is performed sequentially against the driving direction. This means that the driver has full information about the behavior of the predecessor in the next time step.

In order to estimate the adequacy of this approach we briefly discuss the single-lane properties of the sequential

variant of the NaSch model. Using sequential update one obviously cannot ensure translational invariance. E.g. if one performs the update site in a fixed sequence, one generates effectively a bottleneck situation analogous to a system with defect sites. Due to this shortcoming of a site oriented implementation of the update sequence, we implemented the update procedure in the following manner. The lattice update of one time step starts at a "NaSch-car", i.e. a car with  $v = 0$  or  $gap > v$ . In the following the position of all cars are updated car by car against the driving direction<sup>2</sup>.

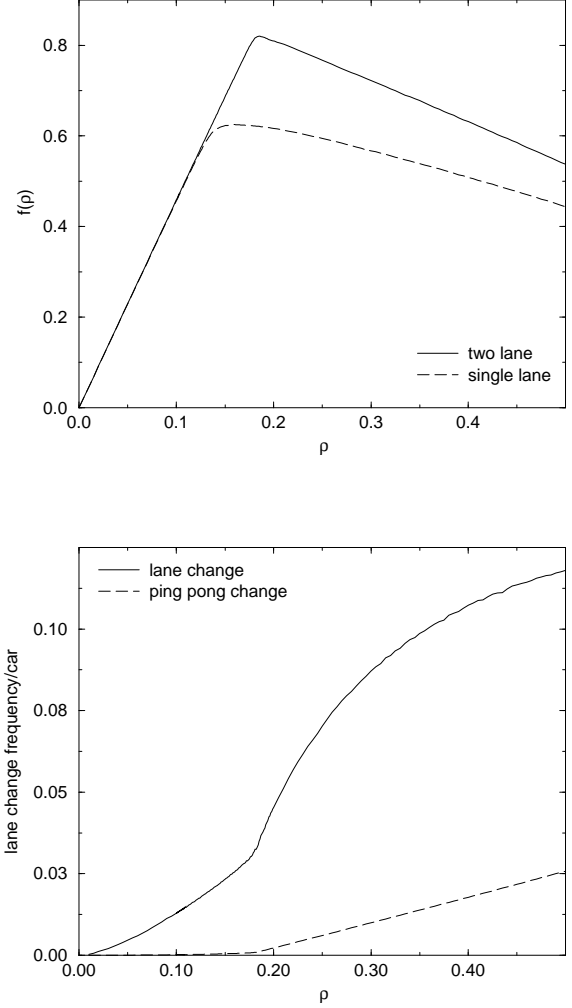


FIG. 10. Top: Fundamental diagram of the single-lane model compared with the two-lane model for systems with  $v_{max} = 5$  and  $p = 0.4$ . Bottom: lane-changing and ping-pong change frequency in the two-lane model.

<sup>2</sup>Using a fixed sequence of cars changes the results only slightly.

Compared to the NaSch model with parallel updating one obtains generically higher values of the average flow, if the same values of  $v_{max}$  and  $p$  are considered. Another feature of the sequential variant is the strong  $p$ -dependence of  $\rho_{max}$ , e.g. in the limit  $p \rightarrow 0$  the maximum value of the flow is reached for  $\rho \approx 1$ . Nevertheless for larger values of the braking noise one obtains quite realistic fundamental diagrams as shown in Fig. 10.

After this short description of the single-lane behavior, we show results for the two-lane system in the presence of particle disorder. Obviously the lane-changing rules can be modified, because the anticipation of the drivers allows for less restrictive safety criteria. Therefore we introduced the following set of lane-changing rules:

- Incentive criterion:

1.  $\Delta v > gap$ , with  $\Delta v = v_{hope} - v_{same}$

- Safety criteria:

1.  $gap_{other} \geq \Delta v$ , with  $\Delta v = v - v_{other}$
2.  $gap_{back} \geq \Delta v$ , with  $\Delta v = v_{back} - v$

Here  $v_{same}$  denotes the velocity of the preceding car on the same lane.  $v_{other}$  and  $v_{back}$  are the velocities of the neighbouring cars on the destination lane.  $gap$ ,  $gap_{other}$  and  $gap_{back}$  have the same meaning as in the definition given in Sec. I.

These lane-changing rules have the same structure as for the parallel model. The first rule is the incentive criterion, which corresponds to an unsatisfactory situation on the origin lane. The safety criteria are necessary for a safe lane-changing maneuver. Obviously the safety criteria differ drastically from those of the parallel case, e.g. if cars at the origin and destination lane move with the same velocity a lane change is possible also for  $gap_{other} = 0$ . These more aggressive lane-changing rules lead to a much more efficient gap usage in the two-lane system. Compared to the single-lane system one obtains an increase of the maximum flow of approximately 30%. Another effect of the modified lane-changing rules is a strongly increased number of lane changes, in particular the number of ping-pong lane changes (i.e. lane changes of the same car in two consecutive time steps). Therefore the fraction of overtakings is reduced. Moreover it is also possible to change the lane at high densities.

In contrast to the parallel model the flow only slightly changes in the presence of one slow car for densities near  $\rho_{max}$  (Fig. 11), while for all other densities the flow of the homogeneous system is recovered. Also the waiting time distribution (Fig. 12) differs only slightly from the homogeneous system, because lane-changing maneuvers can be performed more effectively due to the less restrictive safety criteria.

Finally we investigated the case of 5% slow vehicles in the system, which was sufficient to dominate the system at low densities for the case of parallel update. In contrast to the model introduced by Rickert et al. [12]

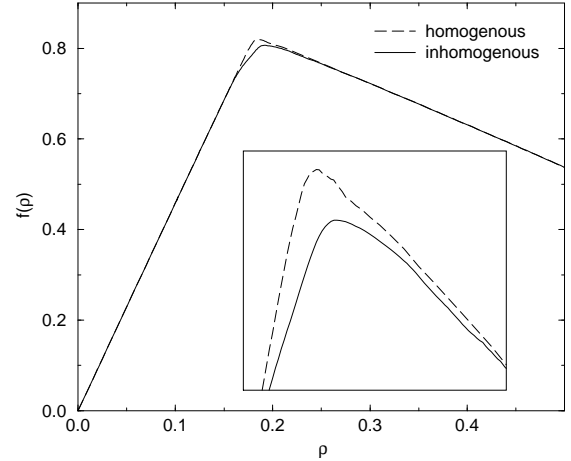


FIG. 11. Flow per lane of the homogeneous and the inhomogeneous system with one slow car for the sequential model.

only small deviations from the homogeneous case can be observed (Fig. 13).

## B. Anticipation models

Although the smoothing effect of the sequential update seems to be quite satisfactory other features of this approach are clearly unrealistic. Therefore we introduced a model variant with parallel update, where anticipation effects are taken into account. Here smaller distances between consecutive fast cars are made possible by estimating the displacement of the predecessor in the next time step. The minimal movement of the predecessor is given by  $\max(v_{next}, 0)$  where  $v_{next} = \min(v_{pred}, gap_{pred}) - 1$  where  $v_{pred}$  and  $gap_{pred}$  denote the velocity and the gap in front of the predecessor. This knowledge allows the introduction of an effective gap between the cars which is given by  $gap_{eff} = gap + v_{next}$ .

The anticipation leads to a slightly increased value of the maximum flow, but major parts of the fundamental diagram are left unchanged. In particular also for small values of the braking noise the fundamental diagram is still quite close to the original model.

This modified version of the NaSch model allows for a set of lane-changing rules analogous to the sequential case:

- Incentive criterion:

1.  $(v > v_{same} \text{ and } gap < v) \text{ or } (gap_{other} > gap)$

- Safety criteria:

2.  $(v_{other} > v) \text{ or } (gap_{other} > gap)$
3.  $v_{back} \leq gap_{back}$

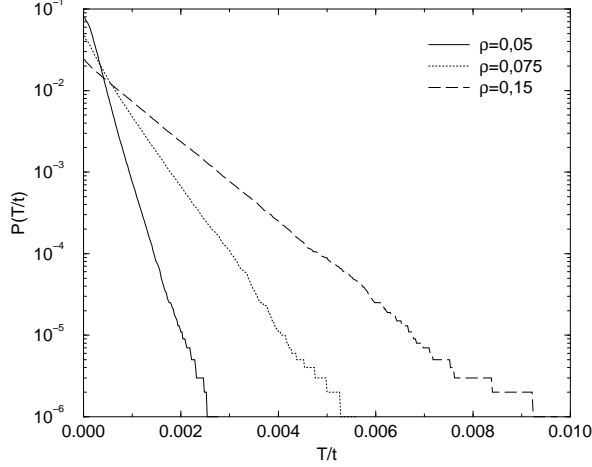


FIG. 12. Waiting time distribution in the sequential model with one slow car.

All quantities have the same meaning as in the definitions given in previous sections. Now lane changes are possible at small distances, e.g. only one empty site between the leading car on the other lane and the changing car is needed if both cars move with the same velocity. Again the minimal gap is determined by velocity differences rather than by the absolute value of the velocity.

The simulation results show that such an implementation of anticipation effects is already sufficient to suppress the drastic influence of slow cars (Fig. 14). Also if 5% of slow cars are included, a reduction of the flow is observable only close to  $\rho_{max}$ . The increased robustness of the modified model shows that anticipation effects are essential for a realistic description of multi-lane traffic.

### C. Other models

We also investigated several model variants which interpolate between the sequential and the anticipation models. We focussed on models where the lanes are updated sequentially, but the update within the lanes is parallel as in the NaSch model. This allows to incorporate lane-changing rules which are more aggressive than those of Rickert et al. [12].

In the simplest case the lane-sequential update is divided into four substeps: a) lane-changes from right to left, b) update of the right lane, c) lane-changes from left to right, d) update of the left lane. These steps can be carried out in different order, e.g. a-b-c-d, b-a-d-c, or a-b-d-c. The first two orderings have the disadvantage that ping-pong lane-changes without forward motion or double updates of cars are possible. The results of the simulations show, however, that there are only small differences of the fundamental diagram and the number of lane changes compared with the parallel update. Fur-

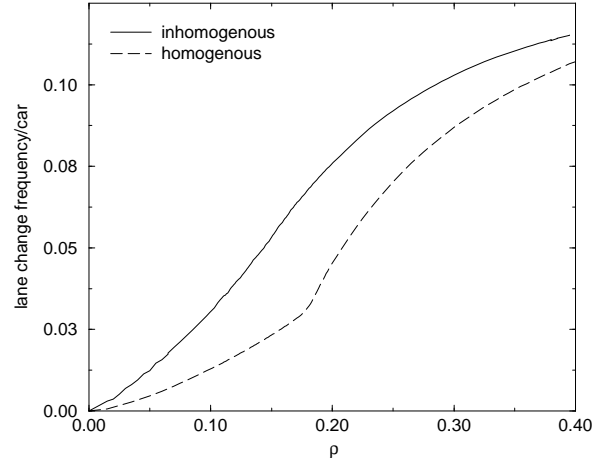
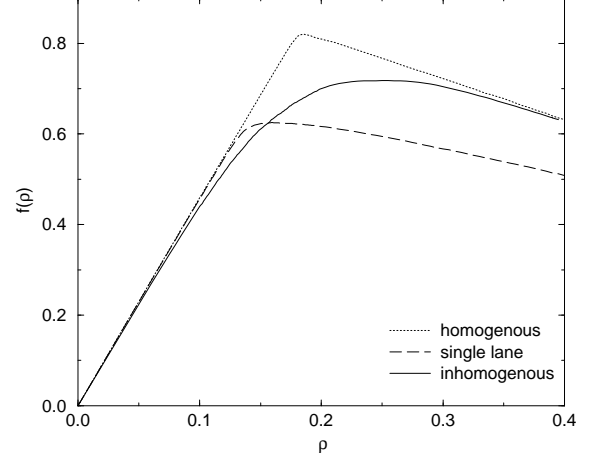


FIG. 13. Top: Comparison of the flow per lane of two-lane systems with a single-lane system for the sequential update. Bottom: Comparison of the lane change frequency of the inhomogeneous two-lane system with the homogeneous system ( $p = 0.4$ ).

thermore the system is still very sensitive to a small fraction of slow cars.

In all of these model variants cars move only ‘side-wards’ during the lane-changing substeps. If one allows cars also to move forward during lane-changes, the situation is improved slightly. Although the cars still need the same safety distances for a lane-change, gaps can be used more efficiently.

Finally, we introduced *temporary* anticipation into the basic model of section II. Cars that changed the lane or the cars directly behind a vehicle that changed to the other lane are allowed to anticipate for the next  $n$  timesteps. Therefore smaller safety distances are possible during lane changes. This reduces the effects of disorder. In comparison to the anticipation model of the previous



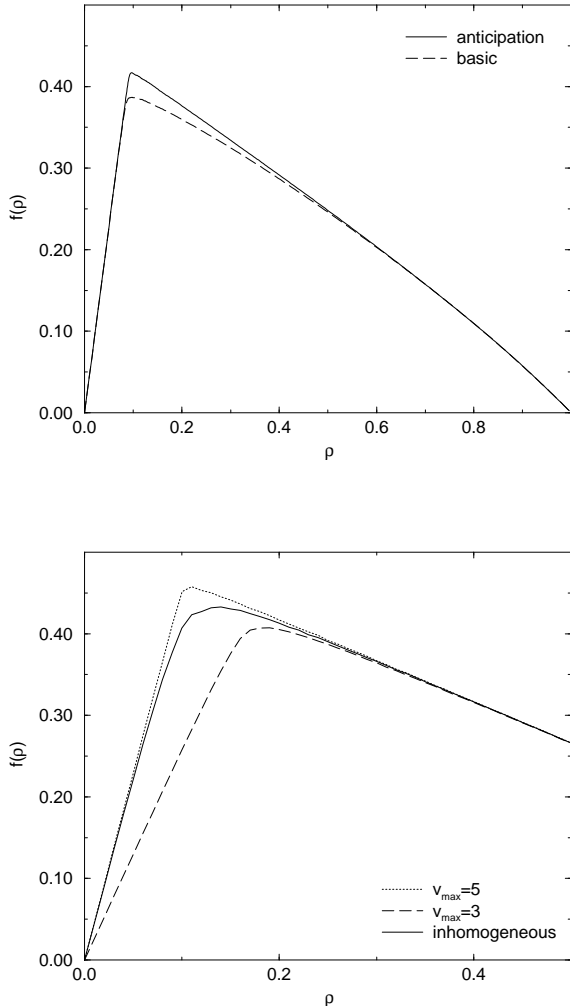


FIG. 14. Top: Comparison of the single-lane “anticipation” and the basic model. Bottom: Average flow of “anticipation” two-lane models ( $p = 0.4$ ).

subsection, the platoons forming behind slower cars are larger and dissolve slower. If the cars stop anticipating after  $n$  timesteps they usually have a short distance to the preceding car and are likely to brake in the next update step due to the deceleration rule of the NaSch model, i.e. temporary anticipation artificially causes jams. In the limit  $n \rightarrow \infty$  one recovers the fundamental diagrams of the anticipation model of the previous subsection. In general, in order for anticipation to be most effective, all cars have to anticipate.

#### IV. SUMMARY AND DISCUSSION

We have shown that even in two-lane systems, where fast cars can overtake slow cars, particle disorder may dominate the behaviour at low densities. The strongest

influence of slow cars has been observed for the NaSch model with symmetric lane-changing rules. For this model variant already one slow car leads to platoon formation on both lanes, i.e. the gap usage at low densities is not very effective. The platoon formation has obviously drastic influence on the performance of the system. If 5% slow cars are included the performance of the heterogeneous system is quite close to a homogeneous system of *slow* cars. For asymmetric lane-changing rules slow cars influence the systems performance less than in the symmetric case, because the density on the “fast” (left) lane is reduced drastically, an effective overtaking is possible. Therefore the flow reduction due to the slow cars has a larger magnitude only close to the maximum.

Although these results look quite realistic, the distribution of cars on both lanes differs drastically from those of real traffic. Measurements on german highways show [19,20] that already for densities below  $\rho_{\max}$  a higher density on the “fast” lane is observable (lane inversion).

Therefore one has to look for model variants which allow for lane changes where the application of less restrictive safety criteria is possible. An important feature of this kind of models probably is the anticipation of the behavior of the predecessor. Via calculating the velocity of the leading car, one can accept much smaller distances than in the NaSch model.

Anticipation is most effectively implemented with a (particle) sequential update against the driving direction. The simulation results show indeed a drastically reduced influence of slow cars, even for symmetric lane-changing rules. Analogous results can be found for a *parallel* model variant where the minimal movement of the predecessor in the next time step is taken into account in order to calculate an effective gap. Compared to the sequential model the single-lane results for this anticipation model are much more realistic and the robustness of the two-lane system against particle disorder is even larger than for the sequential variant.

We also studied the effects of a small fraction of fast cars in a system of slow vehicles. The fundamental diagram is almost identical to that of a pure system of slow cars. The average velocity of the fast cars is larger than that of the slow cars only for small densities.

In conclusion we have shown that particle disorder can lead to platoon formation even in two-lane systems. Having in mind that also non-trivial lattice geometries may have strong effects on the macroscopic behavior of the system, a very careful interpretation of traffic data is necessary in order to distinguish between dynamical, multi-lane, and boundary effects [21].

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